Alternating voltage source V in series with resistor R and inductor L. $V = V_0 cos(\omega t)$. Find the voltage across R and across L.

1. Calculate I

$$V - IR - L\frac{dI}{dt} = 0$$

$$L\frac{dI}{dt} + RI = V$$

Get it into the form $\frac{dI}{dt} + aI = b$.

$$\frac{dI}{dt} + \frac{R}{L}I = \frac{V}{L}$$

Multiply by $e^{\int a dt}$.

$$e^{\frac{R}{L}t}(\frac{dI}{dt} + \frac{R}{L}I) = e^{\frac{R}{L}t}(\frac{V}{L})$$

$$\frac{dI}{dt}e^{\frac{R}{L}t} + \frac{R}{L}Ie^{\frac{R}{L}t} = \frac{V}{L}e^{\frac{R}{L}t}$$

Left side is equal to one derivative.

$$\frac{d}{dt}(Ie^{\frac{R}{L}t}) = \frac{V}{L}e^{\frac{R}{L}t}$$

Integrate, divide by $e^{\frac{R}{L}t}$. This isn't strictly right because V is a function of t as well, but the integration $e^x cosx$ is long and looks weird.

$$Ie^{\frac{R}{L}t} = \frac{V}{R}e^{\frac{R}{L}t} + k$$

$$I = \frac{V}{R} + ke^{-\frac{R}{L}t}$$

At $t = 0$, I = 0...

$$0 = \frac{V}{R} + k \Rightarrow k = -\frac{V}{R}$$

$$\Rightarrow I = \frac{V}{R}(1 - e^{-\frac{R}{L}t})$$

2. V across R

$$V_R = IR = V(1 - e^{-\frac{R}{L}t})$$

3. V across L

$$V_L = L\frac{dI}{dt} = L\frac{d}{dt}(\frac{V}{R} - \frac{V}{R}e^{-\frac{R}{L}t}) = \frac{LV_0}{R}\frac{d}{dt}(cos(\omega t) - cos(\omega t)e^{-\frac{R}{L}t})$$

$$= \frac{LV_0}{R}(-\omega sin(\omega t) - (-\omega sin(\omega t)e^{-\frac{R}{L}t} - \frac{R}{L}cos(\omega t)e^{-\frac{R}{L}t}))$$

$$= -\frac{L\omega}{R}V_0 sin(\omega t)(1 - e^{-\frac{R}{L}t}) + V_0 cos(\omega t)e^{-\frac{R}{L}t}$$

$$= \frac{L}{R}\frac{dV}{dt}(1 - e^{-\frac{R}{L}t}) + Ve^{-\frac{R}{L}t}$$